STAT-1301 ; Lecture 8 ; Feb. 1, '24

Tutoring Centre Review of Probability — Feb. 5 : 12:30–1:20 p.m.

Still in § 4.3.

Ex. (Prob 4-33) An experiment consists of 11 equally likely outcomes denoted by a, b, c, d, e, f, g, h, i, j and k. Consider the events $A = \{b, d, e, j\}$, $B = \{a, c, f, j\}$ and $C = \{c, g, k\}$.

Q'n: Are the events A and B independent?
What about events A and C?

§ 4.3.3 Independent vs. Dependent Events

Def'n: Events A and B are said to be independent if

$$P(A \mid B) = P(A) \quad \text{or} \quad P(B \mid A) = P(B).$$

If $P(A \mid B) \neq P(A)$ or $P(B \mid A) \neq P(B)$ then

A and B are said to be dependent events.

Result (§4.4): Independence of events A and B can also be verified using the following:

If $P(A \cap B) = P(A) \times P(B)$, then A and B are independent events.

Read examples 4_16, 4_17 in ebook.

Ex. (Pr. 4-36; Shoppers data)

One shopper is randomly selected from all 600 shoppers.

i) What is the probability that the randomly selected shopper watches for calories/fat given the shopper is female?

Last Lecture: $P(\text{"Yes"} \mid \text{"Females"}) = \dfrac{106}{300} = 0.353$

ii) Are events "Yes" and "Females" independent?

i.e. $P(\text{"Yes"} \mid \text{"Females"}) \stackrel{?}{=} P(\text{"Yes"})$

$P(\text{"Yes"})$ ($=$ P(a randomly selected shopper watches for Calories/fat)) $= \frac{180}{600} = 0.300$

Since $P(\text{"Yes"} \mid \text{"Females"}) \neq P(\text{"Yes"})$, the events are dependent.

Ex. See problem at Start of Class.

a) Are the events A and B independent? What about events A and C?

Sol'n:

$$P(A \mid B) \stackrel{?}{=} P(A)$$

$$P(A) = \frac{4}{11} \quad \left(= \frac{m}{k} \;;\; \text{Classical Prob.} \right)$$

$$P(A \mid B) = \frac{1}{4}$$

We are Conditioning on event B. This means now $\mathcal{S} = B = \{a, c, f, j\}$. Now, we count how many of the outcomes in A appear in the new sample space; there is only such outcome (j).

$$\underbrace{P(A \mid B)}_{4/11} \neq \underbrace{P(A)}_{1/4} \Rightarrow$$ A and B are dependent events.

$$\underbrace{P(A \mid C)}_{\frac{0}{3}=0} \stackrel{?}{=} \underbrace{P(A)}_{4/11}$$

$\cancel{S} = C = \{c, g, k\}$. None of the outcomes of $A = \{b, d, e, j\}$ occur in C. Since $P(A|C) \neq P(A)$, A & C are dependent.

b) Are the events A and B mutually exclusive?

$$P(A \cap B) \stackrel{?}{=} 0 \quad \Leftrightarrow \quad A \cap B \stackrel{?}{=} \emptyset$$

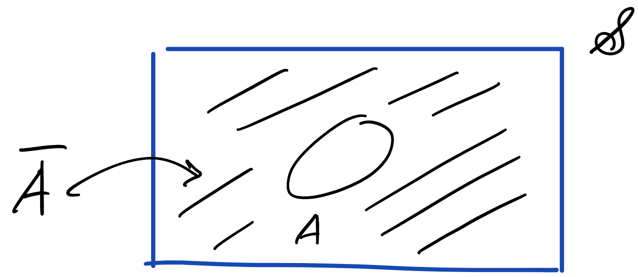
$$A \cap B = \{j\} \neq \emptyset . \quad P(A \cap B) = \frac{1}{11} \neq 0 .$$

A & B are NOT mutually exclusive.

c) Are events A and C mutually exclusive?

$A \cap C = \emptyset \Rightarrow A$ & C are mutually exclusive.

§ 4.3.4 Complement of an Event



Let A be an event. $\bar{A}$ (read as "A Complement") is the set of all outcomes in $\mathcal{S}$ that are not in A.

$$P(A) + P(\bar{A}) = P(\mathcal{S}) = 1$$

$$\boxed{P(\bar{A}) = 1 - P(A)}$$

Given

Ex. Let A be the event that a number less than 3 is obtained when you roll a fair six-sided die once. Find $P(A)$ and $P(\bar{A})$.

Sol'n:

$\mathcal{S} = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 2\}$.

$P(A) = \frac{2}{6} \left(= \frac{m}{k}\right) \to 1/3$.

$P(\bar{A}) = 1 - 1/3 = 2/3$. (Using law of Complements).

$\bar{A} = ?$ $\quad \bar{A} = \{3, 4, 5, 6\}$.

$P(\bar{A}) = \frac{m}{k} = \frac{4}{6} = 2/3$ (using Classical Probability)

Ex. The Probability that a randomly selected UW student did not attend a Stanley Cup Playoff game is 0.88. What is the probability that a randomly selected student attended at least one game?

Sol'n:

Put A = a randomly selected student attended at least one game.

$P(A) = ?$

$$\boxed{P(\text{at least one } \ldots\ldots) = 1 - P(\text{none } \ldots\ldots)}$$

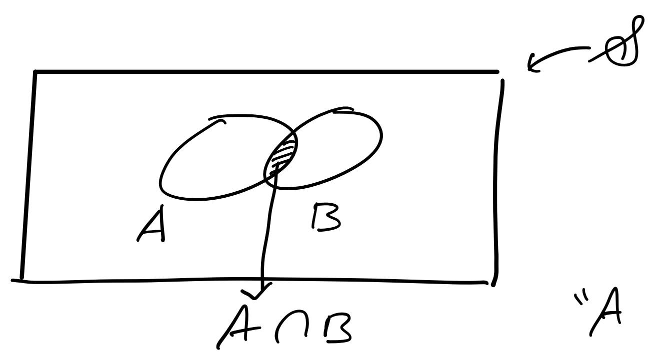
$$P(A) = 1 - P(\bar{A})$$

$= 1 - P($ a randomly selected student attends none of the games $)$

$$= 1 - 0.88 = 0.12$$

§4.4 Intersection of Events & Multiplication Rule.

Recall:



"A intersect B"

$$A \cap B = B \cap A$$

Rule for $P(A \cap B)$:

$$\boxed{P(A \cap B) = P(A) \times P(B \mid A) = P(B) \times P(A \mid B)}$$

Given

Recall: When $A \cap B = \phi$, then $P(A \cap B) = 0$.

Result: Let A and B be independent events, then $P(A \cap B) = P(A) \times P(B)$.

Result: Let $A_1, A_2, \ldots, A_n$ are independent events, then

$$P(A_1 \cap A_2 \cap \ldots \ldots \cap A_n) = P(A_1) \times P(A_2) \times \ldots \times P(A_n).$$

Conditional Probability:

Rearranging the expression for $P(A \cap B)$ gives

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \; ; \; P(B|A) = \frac{P(A \cap B)}{P(A)}.$$

Ex. (Refer to Shoppers example).

Want: $P(\underbrace{\text{"Yes"}}_{A} \mid \underbrace{\text{"Females"}}_{B})$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(\text{"Yes"} \cap \text{"Females"})}{P(\text{"Females"})}$$

$$\hookrightarrow = \frac{106/\cancel{600}}{300/\cancel{600}} = \frac{106}{300}$$

want: $P(\text{"Males"} \mid \text{"No"})$

$$= \frac{P(\text{"Males"} \ \& \ \text{"No"})}{P(\text{"No"})}$$

$$= \frac{168/\cancel{600}}{292/\cancel{600}} = \frac{168}{292}$$ (See last lecture)

Read examples in § 4.4.