STAT_1301; Lecture 21, Apr. 2, '24

Final Exam Date: Friday, Apr. 19, '24 from 1:30 p.m–4:30 p.m.

Final Exam Syllabus: Check my personal website the evening of Apr. 3rd.

Ch. 8

In Ch. 7, we estimated the pop. mean using $\bar{x}$ and the pop. proportion, p, using $\hat{p}$.

$\bar{x}$ and $\hat{p}$ are examples of Point estimators; their observed values are examples of point estimates.

In Ch. 8, we learn how to construct interval estimates for the parameters μ and p.

Def'n: An interval estimate for a pop. parameter provides a range of plausible values for the pop. parameter.

Def'n: A Confidence interval is an interval estimate of the parameter under study; in this course it has the form:

$$\text{point estimate} \pm \text{margin of error}$$

Def'n: The Confidence level associated with a confidence interval states how much confidence we have that our particular interval contains the parameter.

Confidence level is written as $100(1-\alpha)\%$ where α is the level of significance (See STAT_1302).

$\boxed{\sigma \text{ Known}}$

Case I

1. normal pop.
2. $n < 30$

Case II

$n \geq 30$

Case III

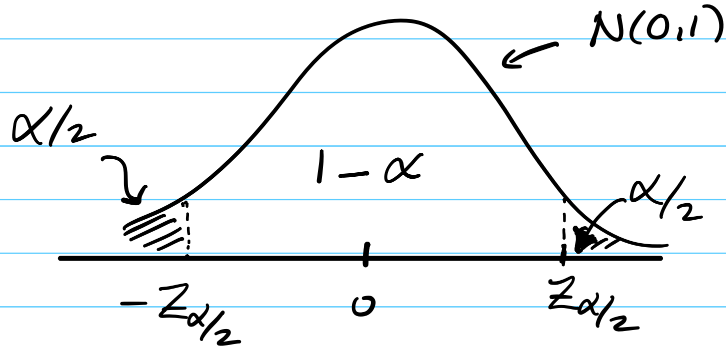
1. pop. not normal
2. $n < 30$

Use $N(0,1)$ for CI Construction

Use nonparametric statistics

Def'n: $Z_{\alpha/2}$ is a critical value defined as that number where $P(Z > Z_{\alpha/2}) = \alpha/2$;

Here, $Z \sim N(0,1)$



§ 8.3 Confidence Intervals for the Pop. mean when the Pop. Std. deviation is Known:

A $100(1-\alpha)\%$ CI for μ when σ is known:

$$\bar{X} \pm Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$ ← Given

where $\bar{x}$ is the sample mean based on a random Sample of Size n, $Z_{\alpha/2}$ is the critical value & σ is the known pop. Std. dev'n.

Ex. A publishing Company has just published a new textbook. The Company wants to estimate the average price of all book similar to its textbook before Setting the price. The research department took a random sample of 25 Comparable textbooks and found Sample mean Price to be \$90.50. It is known that the Standard deviation of the Price of all such textbooks is \$7.50 and the population of Such prices is normal. Construct a 90% Confidence interval for the mean price of all such textbooks.

Sol'n:

$\sigma = 7.50$; X = price ; $n = 25$; $\bar{x} = 90.50$

$$X \sim N(\mu, 7.50).$$

We are in Case I of the flow chart so need normality because $n = 25 < 30$.

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 90.50 \pm 1.65 \frac{7.50}{\sqrt{25}}$$

$$= 90.50 \pm 2.48$$

$$= (88.02\ ,\ 92.98)\ *$$

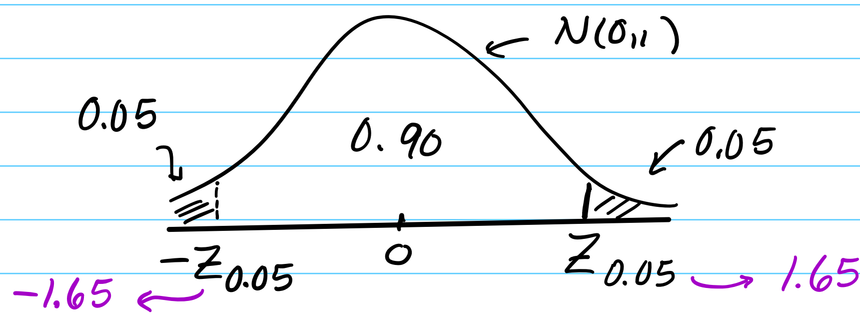


Table IV

90% CI $\Rightarrow$ $100(1-\alpha)\% = 90\%$ $\Rightarrow 1-\alpha = 0.9$

$\Rightarrow \alpha = 0.1 \Rightarrow \alpha/2 = 0.05$

Table IV

z	0.05
-1.6	0.0495

Interpretation: We are 90% confident that the mean price of all such textbooks is between \$88.02 and \$92.98.

Remark: We don't know if the true mean price of all such textbooks is in the interval constructed with 100% certainty, hence we attach the 90% confidence level to our interval.

Meaning of a Confidence Interval:

Upon repeated sampling, for a 90% CI for the mean μ, we expected 90% our intervals to capture/cover the true mean (and 10% do not). For a particular study, the 90% CI may not include μ. Thus, we attach the confidence level.

b) Construct a 95% confidence interval for the mean price of all such textbooks.

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 90.50 \pm 1.96 \times \frac{7.50}{\sqrt{25}} = (**)$$

$95\% = 100(1-\alpha)\% \Rightarrow 1-\alpha = 0.95$; $\alpha = 0.05$;

$\alpha/2 = 0.025$

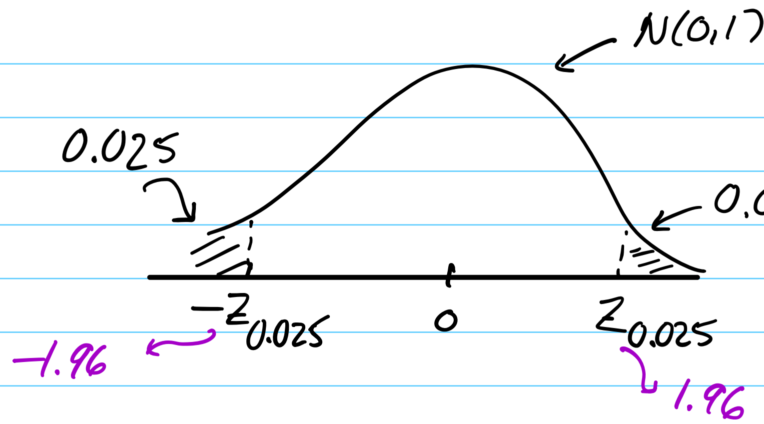


Table IV:

z	0.06
1.9	0.025

(**) $= 90.50 \pm 2.94 = (\$87.56, \$93.44)$

Interpretation: We are 95% Confident that the true mean price of all such textbooks varies between \$87.56 and \$93.44.

c) Find the 99% CI for the mean price.

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$1 - \alpha = 0.99 \; ; \; \alpha = 0.01 \; ; \; \alpha/2 = 0.005$$

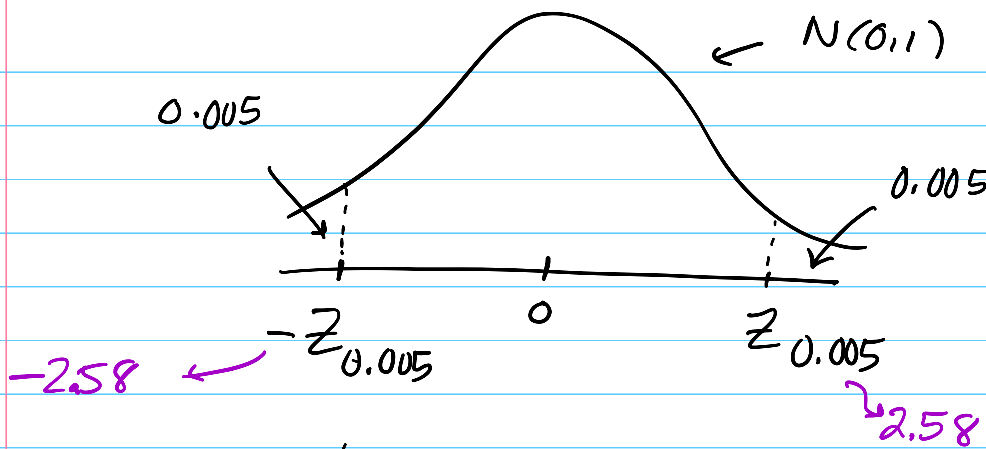
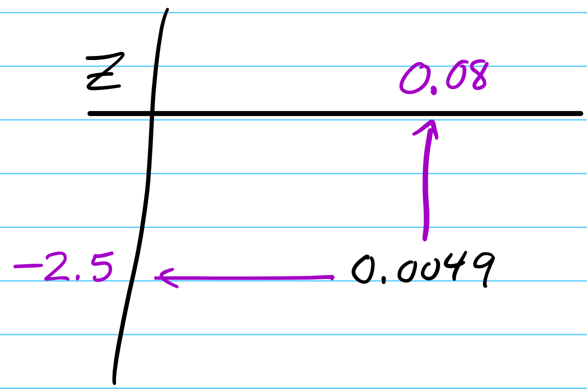


Table IV:



$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 90.50 \pm 2.58 \times \frac{7.50}{\sqrt{25}}$$

$$= 90.50 \pm 3.87$$

$$= (\$86.63 \, , \, \$94.37)$$

We are 99% Confident that the mean price varies between \$86.63 and \$94.37.

Confidence level	Margin of Error	$z_{\alpha/2}$
90%	2.48	1.65
95%	2.94	1.96
99 %	3.87	2.58

Message: As confidence level ↗, so does the margin of error leading to wider confidence intervals.

How can the width of the CI be reduced?

$$m.e. = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

MCQ

i) Increase n

ii) Decrease σ

iii) Reduce the confidence level