STAT-1301; Lecture 19; March 21, '24

In Ch. 6.

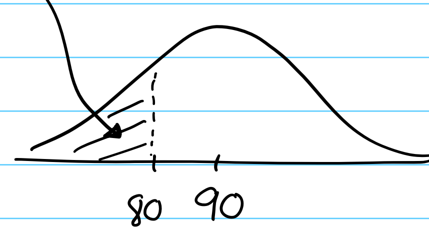
Ex. An aptitude test administered to aircraft Pilot trainees requires a Series of operations to be Performed in quick Succession. Suppose that the time needed to Complete the test is normally distributed with mean $\mu = 90$ minutes and $\sigma = 20$ minutes.

a) To Pass the test, a Candidate must Complete it within 80 minutes. What Percentage of Candidates will pass the test?

Given: X = time needed to Complete the test.

$$X \sim N(90, 20).$$

Want: $P(X < 80)$



$$P(X < 80) = P\left(Z < \frac{80-90}{20}\right) = P(Z < -0.5)$$

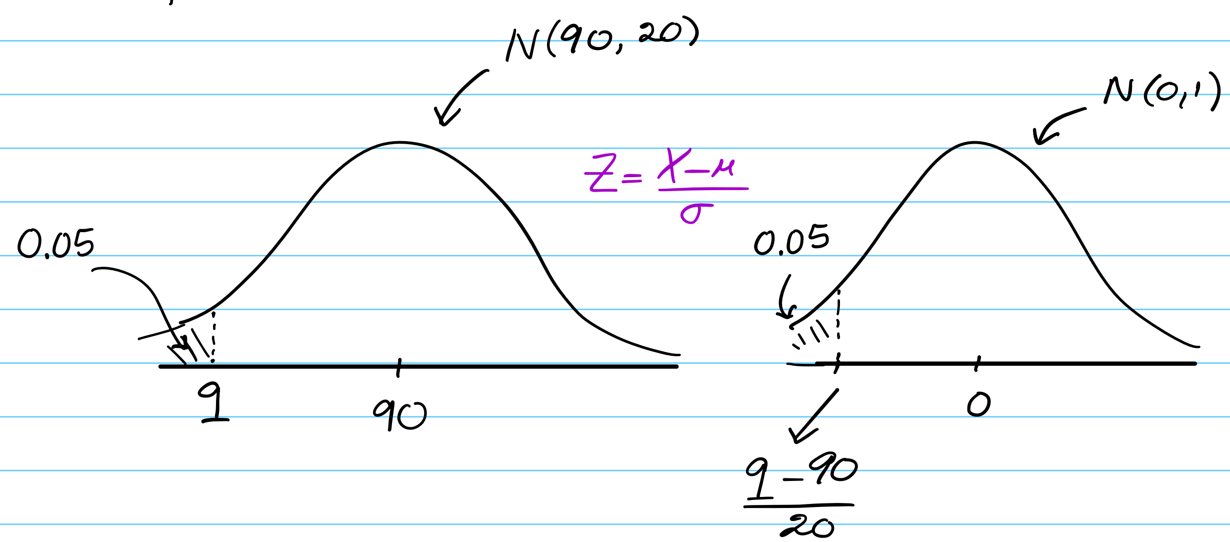
$Z \sim N(0,1)$.

$$= 0.3085$$

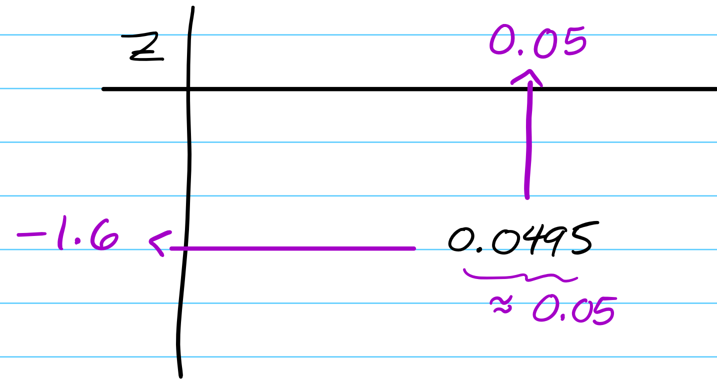
(Table IV).

Approx. 31% of candidates pass the test.

b) If the top 5% of the candidates are to be given a certificate of Commendation, how fast must a candidate complete the test to be eligible for a certificate?



Now this problem involves find the number whose left-tail probability under the standard normal density curve is 0.05.



i.e. $P(Z < -1.65) = 0.05$

Remark: This part b) is asking you to find P_5, the 5^{th} percentile of X.

Now equate the 5^{th} percentile of the $N(0,1)$ distribution to $\frac{q-90}{20}$ and solve for q.

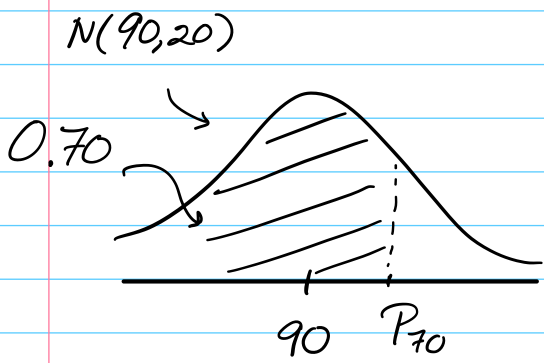
$$\frac{q-90}{20} = -1.65$$

$$q = -1.65(20) + 90 = 57 \text{ minutes}$$

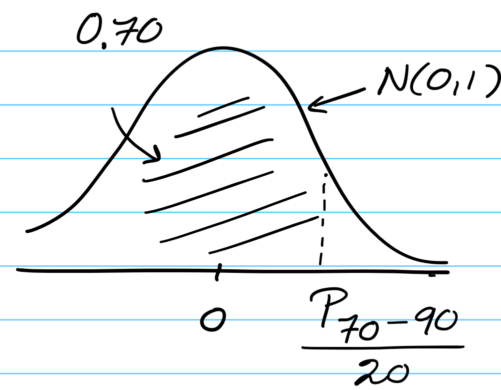
ie. To receive a certificate, an applicant must complete the task in no more than 57 minutes.

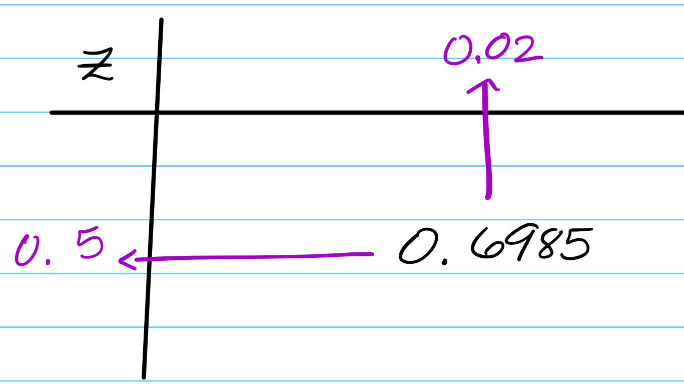
C) Find the 70^{th} percentile of the Probability distribution of Completion times.

Want: P_{70}



$$Z = \frac{X - 90}{20}$$





$$P(Z < 0.52) = 0.6985 \approx 0.70$$

$$\frac{q - 90}{20} = 0.52$$

$$\Rightarrow \quad q = 20(0.52) + 90 = 100.4 \text{ minutes}$$

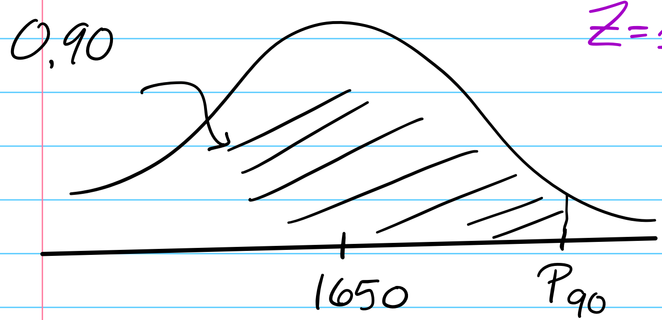
≈ 70% of Completion times are 100.4 minutes or less.

Ex (Pr. 6.42) See ebook.

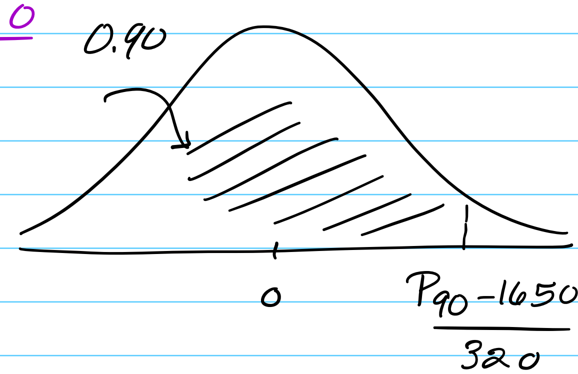
X = monthly electric consumption per household

$X \sim N(1650, 320)$

want: P_{90}



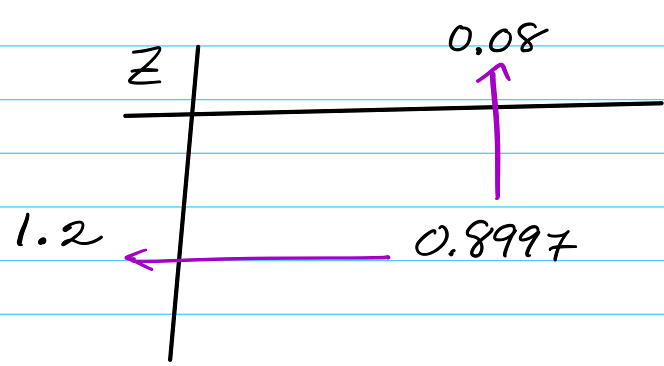
$Z = \frac{X - 1650}{320}$



Look up 0.90 in the area portion of Table IV.

(This gives us the 90^{th} percentile of the $N(0,1)$ distribution.) We get 1.28.

ie. $P(Z < 1.28) = 0.8997 \approx 0.90$



$$\frac{q - 1650}{320} = 1.28$$

$$q = 320(1.28) + 1650 = 2059.6 \text{ Kwh}$$

Bill J's monthly electric consumption is 2,059.6 kwh.

End of Ch. 6.

Ch. 7 now.

See pdf notes posted on Nexus: "Ch. 7".

Ex. A package of bulbs claims that these bulbs have an average life of 24,966 hours. Assume that the lives of all such bulbs have a normal distribution with a mean of 24,966 hours and a standard deviation of 2,000 hours. Find the probability that the mean life of a random sample of 25 such bulbs is within 650 hours of the population mean.

Sol'n:

X = lifetime ; $X \sim N(24966, 2000)$

Want:

$P(24966 - 650 < \bar{X} < 24966 + 650)$

$= P(24316 < \bar{X} < 25616)$