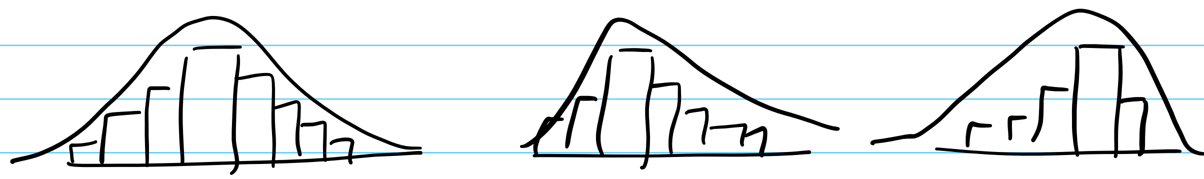
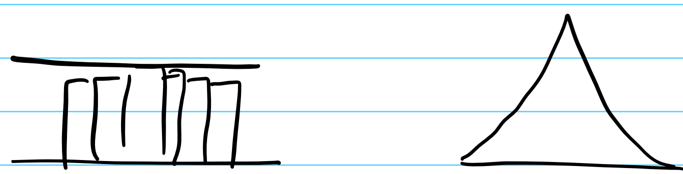
STAT 1301; Lecture 17; π-day, 2024!

Ch. 6.



X: quantitative variable



etc.

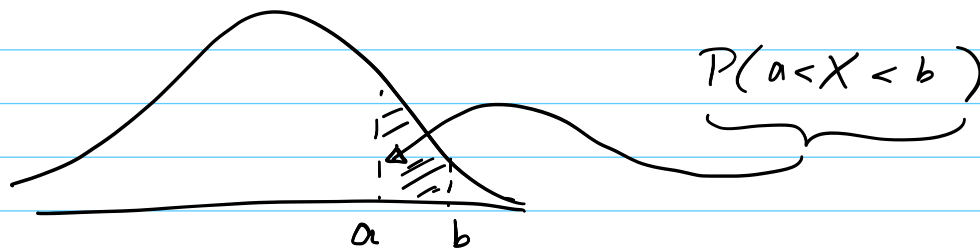
These smoothed functions are known as

Probability density functions.

Message: For continuous r.v.'s events are of the form $P(a < X < b)$. [c.f. discrete r.v.'s, events are of the form $P(X=a)$, $P(a \le X \le b) =$

$P(X=a) + \dots + P(X=b)$].

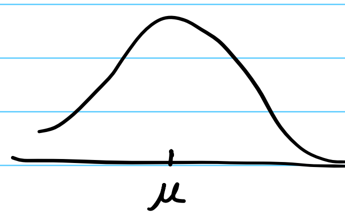
$\therefore X$ continuous, $P(a < X < b)$ means find the area under the probability density function curve over (a, b).



§ 6.2 The Normal Distribution

(i.e. the bell_shaped distribution in Ch. 3)

Facts:



- its probability density equation is symmetric about some number μ.

- $X \sim N(\mu, \sigma)$ read as X follows a normal distribution with population mean μ and population standard deviation σ.

- Some texts: $X \sim N(\mu, \sigma^2)$

$\sigma^2 \rightarrow$ Pop. Variance

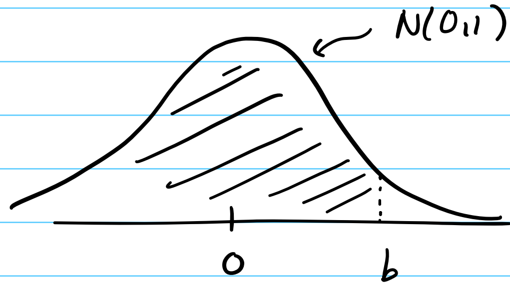
- If X is Normal with mean $\mu = 0$ and $\sigma = 1$, then use Z instead of X and write

$$Z \sim N(0,1). \leftarrow \text{Standard normal distribution.}$$

- We use Table IV (App. B) to obtain left-tail probabilities of the $N(0,1)$ distribution.

ie. $Z \sim N(0,1)$. want: $P(Z < b)$

$\Leftrightarrow$



Ex. $Z \sim N(0,1)$. $P(Z < -1.95) = ?$

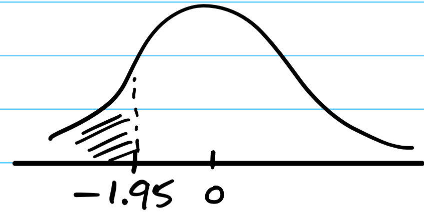
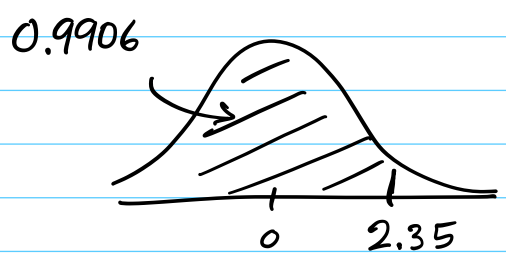


Table IV

z	0.05
-1.9	$\boxed{0.0256}$

$P(Z < -1.95) = 0.0256.$

Ex. $Z \sim N(0,1)$. $P(Z < 2.35)$.



z	0.05
2.3	$\boxed{0.9906}$

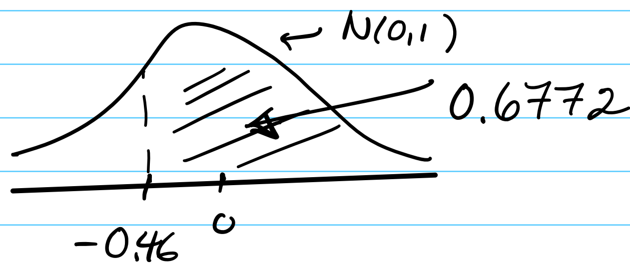
Ex. $Z \sim N(0,1)$. $P(Z > -0.46)$

(i.e. find the right-tail probability of -0.46)

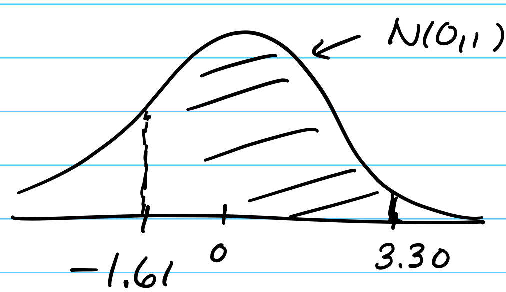
z	0.06
-0.4	$\boxed{0.3228}$

$\therefore P(Z > -0.46) = 1 - P(Z < -0.46)$

$= 1 - 0.3228 = 0.6772$



Ex. $Z \sim N(0,1)$. $P(-1.61 < Z < 3.30) = ?$



$P(-1.61 < Z < 3.30) = P(Z < 3.30) - P(Z < -1.61)$

$= 0.9995 - 0.0537$

$= 0.9458$ (Homework !)

Ex. Verify the empirical rule for $Z \sim N(0,1)$.

i) $P(-1 < Z < 1) \overset{?}{=} 0.68$

ii) $P(-2 < Z < 2) \overset{?}{=} 0.95$

iii) $P(-3 < Z < 3) \overset{?}{=} 0.997$

i) $P(-1 < Z < 1) = P(Z < 1) - P(Z < -1)$

$= 0.8413 - 0.1587 = 0.6826$

≈ 0.68

Ex. $X \sim N(\mu, \sigma)$. Verify the Empirical Rule.

$P(\mu - \sigma < X < \mu + \sigma) \overset{?}{=} 0.68$

$P(\mu - 2\sigma < X < \mu + 2\sigma) \overset{?}{=} 0.95$

$P(\mu - 3\sigma < X < \mu + 3\sigma) \overset{?}{=} 0.997.$

$P(\mu - \sigma < X < \mu + \sigma)$

TBC ....

Result: If $X \sim N(\mu, \sigma)$, then

$$Z = \frac{X - \mu}{\sigma} \sim N(0, 1).$$

Z-score

"Standardized X".

Extra: $X \sim N(\mu, \sigma)$. want to show that

$$P(\mu - \sigma < X < \mu + \sigma) \cong 0.68$$

$$\rightarrow P(\mu - \sigma < X < \mu + \sigma)$$

$$= P(\mu - \sigma - \mu < X - \mu < \mu + \sigma - \mu)$$

$$= P(-\sigma < X - \mu < \sigma)$$

$$= P\left(\frac{-\sigma}{\sigma} < \frac{X-\mu}{\sigma} < \frac{\sigma}{\sigma}\right)$$

$$= P(-1 < Z < 1)$$

$$\cong 0.68$$

Ex. $X \sim N(50, 10)$; $\sigma = 10$.

$$P(X < 55) = \ ?$$

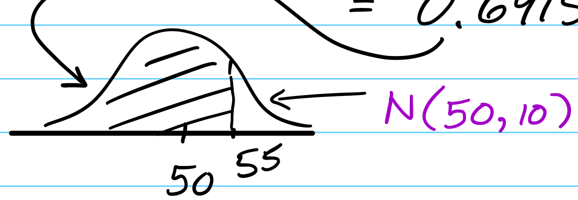
$$P(X < 55) = P\left(\frac{X - 50}{10} < \frac{55 - 50}{10}\right)$$

$$= P(Z < 0.5) \qquad Z \sim N(0,1).$$

$$= 0.6915$$

from Table IV

direct lookup



Ex. The average monthly mortgage payment for all homeowners in a city is \$2,850. Suppose the distribution of monthly mortgages is normal with a mean of \$2,850 and a standard deviation of \$420. Find the probability that a randomly selected homeowner pays a monthly mortgage that is

i) less than \$1200

Given: X = monthly mortgage payment.

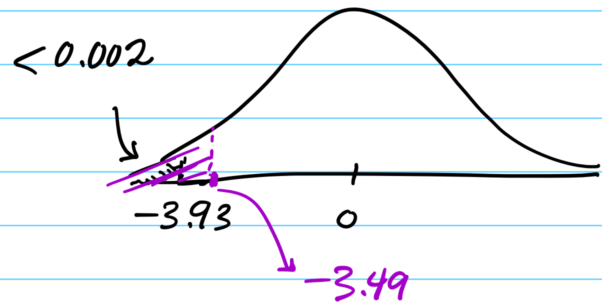
$X \sim N(2850,\ 420)$

$P(X < 1200) = ?$

$$P(X < 1200) = P\left(Z < \frac{1200 - 2850}{420}\right)$$

$$= P(Z < -3.93) < 0.0002$$

(≈ 0.0002)



Interpretation:

a) The Prob. that a randomly selected homeowner pays less than \$1200/month is approx. 0.0002.

b) The percentage of homeowners whose monthly mortgage payment is less than \$1200 is approx. 0.02%.

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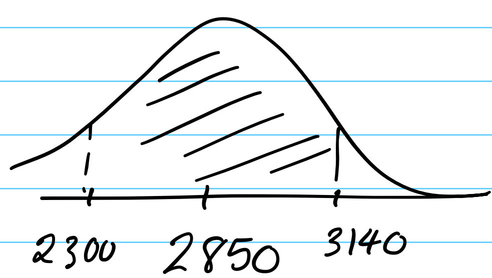
b) between \$2,300 and \$3,140.



Want: $P(2300 < X < 3140)$

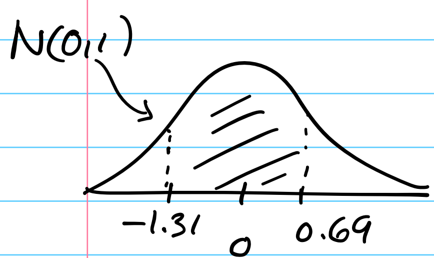
$$P(2300 < X < 3140) = P\left(\frac{2300-2850}{420} < Z < \frac{3140-2850}{420}\right)$$

$$= P(-1.31 < Z < 0.69)$$

$$= P(Z < 0.69) - P(Z < -1.31)$$

$$= 0.7549 - 0.0951 = 0.6598$$

(See Table IV).



≈ 66% of homeowners' monthly mortgage payments are between \$2,300 and \$3,140.

iii) more than \$3,600

Want: $P(X > 3600)$.

$$P(X > 3600) = P\left(Z > \frac{3600-2850}{420}\right)$$

$$= P(Z > 1.79)$$

$$= 1 - P(Z < 1.79)$$

$$= 1 - 0.9633 = 0.0367 \text{ (Check!)}$$

Table IV

Approx. 3.7% of monthly mortgage payments exceed $3,600.