STAT-1301; Lecture 16; March 12, '24

Ex. At a bank, past data show that 8% of all credit card holders default at some time in their lives. On one recent day, this bank issued 12 credit cards to new customers. Find the probability that of these 12 new customers, eventually

a) exactly one will default

Let X = # of customer out of 12 who will default

$$X \sim Bin(12, 0.08)$$

$$P(1) = {}_{12}C_1 \times 0.08^1 \times 0.92^{11} = \frac{12!}{1!\,11!} \times 0.08 \times 0.92^{11}$$

$$= 12 \times 0.08 \times 0.92^{11}$$

$$= 0.384.$$

b) at least one will default.

$$P(X \geq 1) = P(1) + P(2) + \cdots + P(12)$$

$$= 1 - P(X < 1)$$ law of complements

$$= 1 - P(0)$$

$$= 1 - {}_{12}C_0 \; 0.08^0 \times 0.92^{12}$$

$$= 1 - \frac{\cancel{12!}}{0!\,\cancel{12!}} \times 1 \times 0.92^{12} = 1 - 0.368 =$$

$$= 0.632.$$

Ex. An average of 6.3 robberies occur per day in a large city.

a) Find the probability that on any given day, exactly 3 robberies will occur in this city.

Given: X = # of robberies per day

$X \sim \text{Poisson}(6.3)$

Want: $P(3) = e^{-6.3} \dfrac{6.3^3}{3!} = 0.0765$

b) Find the probability that at least 3 robberies occur on any given day in this city.

$$P(X \geq 3) = P(3) + P(4) + \cdots$$

$$= 1 - P(X < 3) \quad \text{law of Complements}$$

$$= 1 - [P(0) + P(1) + P(2)].$$

$$= 1 - \left[e^{-6.3} \frac{6.3^0}{0!} + e^{-6.3} \frac{6.3^1}{1!} + e^{-6.3} \frac{6.3^2}{2!} \right]$$

$$= 1 - e^{-6.3} \left[1 + 6.3 + \frac{6.3^2}{2} \right]$$

$$= 1 - 0.0498 = 0.952.$$

c) Find the Probability that on any given hour, the number of robberies in this city will be exactly one.

X = # of robberies per day ;

$X \sim \text{Poisson}(6.3)$.

Let Y = # of robberies Per hour

$Y \sim \text{Poisson}(0.3 \quad)$

6.3 robberies per day means $\frac{6.3}{24} = 0.3$ robberies per hour

want: $P(Y=1)$, $Y \sim \text{Poisson}(0.3)$

$$P(Y=1) = e^{-0.3} \frac{0.3^1}{1!} = 0.222$$

Ex. Based on the analysis of the future demand for its products, the financial department at a corporation has determined that there is a 0.17 probability that the company will lose \$1.2 million during the next year, a 0.21 probability that it will lose \$0.7 million during the next year, a 0.37 Probability that it will make a profit of \$0.9 million, and a 0.25 Probability that it will make a \$2.3 million Profit during the next year.

a) Let X be the random variable that denotes the profit earned by this Corporation during the next year. Write down the Probability distribution of X.

x	-1.2	-0.7	0.9	2.3
$P(x)$	0.17	0.21	0.37	0.25

$\sum P(x) = 1$

b) Find the mean and Standard deviation of the random variable X in part (a).

$$\mu = \sum x P(x)$$

$$= -1.2(0.17) + -0.7(0.21) + 0.9(0.37) + 2.3(0.25)$$

$$= 0.557$$

$$\sigma^2 = \sum x^2 P(x) - \mu^2$$

$$= (-1.2)^2(0.17) + (-0.7)^2(0.21) + 0.9^2 \times 0.37 + 2.3^2(0.25) - 0.557^2$$

$$= 1.9699$$

$$\therefore \quad \sigma = \sqrt{1.9699} = 1.66$$

c) Give a brief interpretation of the mean in part (b).

The Corporation can expect to make $0.557 million in profit during the next year.

End of Ch. 5.

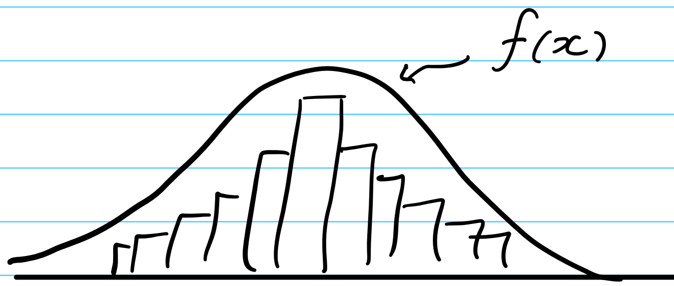
Test 2 covers Ch. 5 & 6.

Ch. 6. Continuous Random Variables

A Continuous r.v. is a r.v. that takes on values over an interval of the real line.

eg. height, weight, IQ, wait times for Surgery, income, test Scores.

The Probability distribution of a Continuous random variable is the Smoothed relative frequency distribution histogram.



$f(x)$ is known as the Probability density function of the r.v. X. It gives us all the info. needed to compute $P(a < X < b)$.

Properties of a Probability Density Function :

i) $f(x) \geq 0$

ii) The entire area under $f(x)$ equals one.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

(We don't use $f(x)$ directly in STAT 1301).

Facts : X is Continuous $\Rightarrow$

$P(a \leq X \leq b) \Leftrightarrow$ Area under $f(x)$ where x lies in $[a, b]$.

$\Leftrightarrow$ The Probability that a randomly selected member from this population has values between "a" and "b".

$\Leftrightarrow$ The percentage of the Population whose X value lies in $[a, b]$.

Fact: $P(a \le X \le b) = P(a < X \le b)$

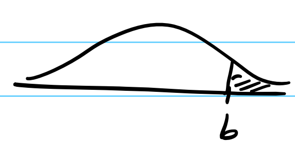
$= P(a \le X < b)$

$= P(a < X < b)$;

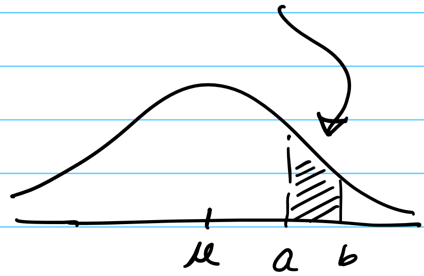
for X a continuous r.v.

Fact: X continuous $\Rightarrow P(X = a) = 0$.

Fact: $P(X > b) = 1 - P(X < b)$



Fact: $P(a < X < b) = \underbrace{P(X < b)}_{\text{entire area under } f(x) \text{ to the left of } b} - \underbrace{P(X < a)}_{\text{entire area under } f(x) \text{ to the left of } a.}$



[Note: $P(a < X < b) = \int_a^b f(x)\,dx$ (Calc. II).

We will not use calculus. We will use

Table IV (App. B) to determine $P(X < b)$.