STAT-1301; Lecture 15; March 7, '24

Ex. The probability that a person who undergoes a kidney operation is 0.6. Find the Probability that of the six patients who undergo similar operations:

a) none will recover

Let X = # of patients out of six who recover

want: $P(X=0)$

i) $n=6$ trials

ii) Bernoulli trials: Two possible outcomes which are recover/do not recover.

iii) Trials independent: Yes, because $\frac{n}{N} < 0.05$

iv) $P(S) = P(\text{recovers}) = 0.6$ for all trials.

$X \sim Bin(6, 0.6)$

* Can say independence due to biological reasons.

$$P(X=0) = {}_6C_0 \times 0.6^0 \times 0.4^{6-0}$$

$$= \frac{6!}{0!(6-0)!} \times 1 \times 0.4^6$$

$$= \frac{6!}{0! \times 6!} \times 1 \times 0.4^6$$

$0! = 1$

$$= 0.4^6 = 0.0041.$$

b) all will recover

want: $P(X=6) = P(\text{all recover})$

$$P(X=6) = {}_6C_6 \; 0.6^6 \times 0.6^{6-6}$$

$$= \frac{6!}{6!(6-6)!} \times 0.6^6 \times 0.6^0$$

$0.6^0 = 1$

$$= \frac{6!}{6!\,0!} \times 0.6^6 \times 1$$

$0! = 1$

$$= 0.6^6 = 0.0467$$

c) half will recover

want: $P(X=3)$

$$P(X=3) = {}_6C_3 \; 0.6^3 \times 0.4^{6-3}$$

$$= \frac{6!}{3!(6-3)!} \times 0.6^3 \times 0.4^3$$

$$= \frac{6!}{3! \times 3!} \times 0.6^3 \times 0.4^3$$

$$= 20 \times 0.6^3 \times 0.4^3 = 0.2765$$

$n=6$, $p=0.6 = P(\text{recover})$, $q = 1-p = 0.4 = P(\text{not recover})$

d) at least half will recover.

$$P(X \geq 3) = P(X=3 \text{ or } X=4 \text{ or } X=5 \text{ or } X=6)$$

$$= P(X=3) + P(X=4) + P(X=5) + P(X=6)$$

↳

$$= 0.2765 + {}_6C_4 \; 0.6^4 \times 0.4^{6-4} + {}_6C_5 \; 0.6^5 \times 0.4^{6-5} + 0.0467$$

$$= 0.2765 + 15(0.0207) + 6(0.0311) + 0.0467$$

Check! Check!