STAT-1301; Lecture 14; March 5, '24

See Nexus for Assignment #4; Due March 14, '24

In Ch. 5 now.

X is a discrete r.v. with Probability distribution, $P(x)$.

Last Class: the mean of a discrete r.v. is

$$\mu = E(X) = \sum_{\text{all } x} x P(x)$$

Ex. Refer to the Consumer Preferences example.

X = # of Consumers who prefer Product A out of 4 Consumers

x	0	1	2	3	4
$P(x)$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

What is the expected number of consumers who prefer product A ?

$$\mu = E(X) = \sum_{\text{all } x} x P(x) =$$

$$\to = 0 \times \frac{1}{16} + 1 \times \frac{4}{16} + 2 \times \frac{6}{16} + 3 \times \frac{4}{16} + 4 \times \frac{1}{16}$$

$$= 2.$$

Ex. A store manager determines the Probability distribution for the number of brand A batteries sold per week to be the following:

x	0	1	2	3	4	5
$P(x)$	0.1	0.1	0.2	k	0.2	0.1

a) What is $P(3)$?

Since $\sum_{x=0}^{5} P(x) = 1$, $P(3) = 1 - (0.1 + 0.1 + 0.2 + 0.2 + 0.1)$

$= 1 - 0.7$

$= 0.3$

b) Find the mean number of brand A batteries sold per week.

$$\mu = \sum_{\text{all } x} x P(x) = 0 \times 0.1 + 1(0.1) + 2(0.2) + 3(0.3) + 4(0.2) + 5(0.1) = 2.7$$

Variance of a Discrete R.V. X:

$$\sigma^2 = Var(X) \overset{def.}{=} E(X-\mu)^2 \quad \text{where}$$

long-way

$$\mu = E(X) = \sum_{\text{all } x} x P(x)$$

Use Short-cut formula:

$$\sigma^2 = E(X^2) - \mu^2$$ Given on test/final

where

$$E(X^2) = \sum_{\text{all } x} x^2 \cdot P(x)$$

Ex. Refer to the battery example.

Find the variance of X where X is the number of brand A batteries sold per week.

$$\sigma^2 = \sum_{\text{all } x} x^2 P(x) - \mu^2$$

From above, $\mu = 2.7$

$$\sum_{\text{all } x} x^2 P(x) = 0^2(0.1) + 1^2(0.1) + 2^2(0.2) +$$

$$+ 3^2(0.3) + 4^2(0.2) + 5^2(0.1) = 9.3.$$

$$\sigma^2 = \sum x^2 P(x) - \mu^2 = 9.3 - 2.7^2 = 2.01$$

The Standard deviation of the r.v. X is

$$\sigma = \sqrt{Var(X)}$$

In this example, $\sigma = \sqrt{2.01} = 1.418 \approx 1.42$.

Now See PDF on Binomial Distribution (§ 5.4).